Model Answer

Set- I

Std. – 12th

Subject-Maths

Section A

Q.1 Select and write the correct answer

(i) Answer: **b)** $\sim p \vee \sim q$

Since $p \vee q$ is true and $p \wedge q$ is false, one of p or q is true and other is false.

(ii) Answer: **c)** $\frac{\pi}{3}, \frac{4\pi}{3}$

$tanx = \sqrt{3}, \therefore tanx = tan\frac{\pi}{3} \& tanx = tan\left(\pi + \frac{\pi}{3}\right) \therefore x = \frac{\pi}{3}, \frac{4\pi}{3}$

(iii) Answer: **c)** $\frac{\mathbf{11}}{\sqrt{\mathbf{238}}}$

Here, $a_1 = 2, b_1 = 2, c_1 = -3, a_2 = 1, b_2 = -2, c_2 = 3$

$cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right| = \left|\frac{2-4-9}{\sqrt{4+4+9}\sqrt{1+4+9}}\right|$

$= \frac{11}{\sqrt{17}\sqrt{14}} = \frac{\mathbf{11}}{\sqrt{\mathbf{238}}}$

(iv) Answer: **b)** 2

$\begin{vmatrix} 1 & -2 & 1 \\ a & -5 & 3 \\ 5 & -9 & 4 \end{vmatrix} = 0$

$\therefore 1(-20 + 27) + 2(4a - 15) + 1(-9a + 25) = 0$

$\therefore a = 2$

(v) Answer: **c)** $\frac{\mathbf{1}}{\boldsymbol{x}(\mathbf{2}\boldsymbol{y}-\mathbf{1})}$

$y = \sqrt{logx + \sqrt{logx + \sqrt{logx + \cdots . \infty}}}, \therefore y^2 = logx + y,$

$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}, \therefore \frac{dy}{dx} = \frac{\mathbf{1}}{\boldsymbol{x}(\mathbf{2}\boldsymbol{y}-\mathbf{1})}$

(vi) Answer: **c)** $\mathbf{4.0125}$

$f(a + h) = \sqrt{16.1}, a = 16 \text{ and } h = 0.1, f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}, f(a) = f(16) = \sqrt{16} = 4$

$f'(a) = f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

$f(a + h) \approx f(a) + hf'(a)$

$\therefore \sqrt{16.1} \approx 4 + (0.1) \times \frac{1}{8} = \mathbf{4.0125}$

(vii) Answer: **c)** $\frac{\boldsymbol{\pi}}{\mathbf{2}} - \mathbf{1}$

$\int_0^{\frac{\pi}{2}} xcosx\, dx = [xsinx - \int 1. sinxdx]_0^{\frac{\pi}{2}}$

$[xsinx + cosx]_0^{\frac{\pi}{2}} = \frac{\boldsymbol{\pi}}{\mathbf{2}} - \mathbf{1}$

(viii) **Answer: b)** $\boldsymbol{y = xlogx - x + c}$

$e^{\frac{dy}{dx}} = x, \therefore \frac{dy}{dx} = logx, dy = logxdx$

Integrating, $y = xlogx - \int \left[\frac{d}{dx}(logx) \int 1dx\right] dx + c, \therefore y = xlogx - x + c$

Q.2 Answer the following

(i) $P(-\sqrt{3}, 1) = \left(\mathbf{2}, \frac{\mathbf{5\pi}}{\mathbf{6}}\right)$

(ii) Separate equations are $y = 2x\ and\ y = -3x$

Joint equation is $(2x - y)(3x + y) = 0, \therefore \mathbf{6x^2 - xy - y^2 = 0}$

(iii) $y = [acos^3x + bsin^3x]^3$

Differentiating w.r.t.x

$\frac{dy}{dx} = 3[acos^3x + bsin^3x]^2 \frac{d}{dx}[acos^3x + bsin^3x]$

$= 3[acos^3x + bsin^3x]^2[3acos^2x(-sinx) + 3bsin^2x(cosx)]$

$= \mathbf{9[acos^3x + bsin^3x]^2[bsin^2xcosx - acos^2xsinx]}$

(iv) **Order is 2 and degree is 1**

Section B

Attempt any Eight

Q.3 Negation

(a)There exists a natural number which is an integer.

(b)$\exists n \in N$,such that $n + 1 \le 2$

Q.4 $x + 2y + z = 6, 3x - y + 3z = 10, 5x + 5y - 4z = 3$

Matrix equation is

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 3 \\ 5 & 5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1, R_3 \to R_3 - 5R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -4 & 0 \\ 0 & 0 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -27 \end{bmatrix}$$

The equations are $x + y + z = 6$ ... (1)

$-4y = -8, \therefore y = 2, -9z = -27, \therefore z = 3$

In (1), $x + 2 + 3 = 6, \therefore x = 1$

$\therefore \mathbf{x = 1, y = 2, z = 3}$

Q.5 Using $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

L.H.S.= $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\cdot\frac{2}{11}}\right) = \tan^{-1}\left(\frac{11+4}{22-2}\right) = \tan^{-1}\left(\frac{3}{4}\right) =$

$R.H.S$

Q.6 Consider $3x^2 + kxy + 2y^2 = 0$

Divide by $x^2, 3 + k\frac{y}{x} + 2\frac{y^2}{x^2} = 0$….(i)

Since lines pass through the origin, $\frac{y}{x}$ = m,slope of the lines.

If 2x + y = 0 is one of the lines then m = -2 must satisfy equation (i)

$\therefore 3 + k(-2) + 2(4) = 0, \mathbf{k = \frac{11}{2}}$

Q.7 Equation of the line is

$3x + 1 = 6y - 2 = 1 - z$

$\therefore 3\left(x + \frac{1}{3}\right) = 6\left(y - \frac{1}{3}\right) = -(z - 1)$

$\therefore \frac{3\left(x+\frac{1}{3}\right)}{6} = \frac{6\left(y-\frac{1}{3}\right)}{6} = \frac{-(z-1)}{6}$

$\therefore \frac{\left(x+\frac{1}{3}\right)}{2} = \frac{\left(y-\frac{1}{3}\right)}{1} = \frac{(z-1)}{-6}$

$\therefore$The line passes through the point $\left(-\frac{1}{3}, \frac{1}{3}, 1\right)$ and its drs are $2, 1, -6$

$\therefore$ Vector equation is

$\boldsymbol{\bar{r} = \left(-\frac{1}{3}\bar{\imath} + \frac{1}{3}\bar{\jmath} + \bar{k}\right) + \lambda(2\bar{\imath} + \bar{\jmath} - 6\bar{k})}$

Q.8 $cos3x = \frac{1}{\sqrt{2}}, \therefore cos3x = cos\frac{\pi}{4}$

We know, if $cosx = cos\alpha$,then $x = 2n\pi \pm \alpha, n \in Z$

$\therefore 3x = 2n\pi \pm \frac{\pi}{4}, \therefore \boldsymbol{x = \frac{2n\pi}{3} \pm \frac{\pi}{12}, n \in Z}$

Q.9 f(x) $= 2x^3 - 15x^2 - 84x - 7$

$\therefore f'(x) = 6x^2 - 30x - 84 = 6(x^2 - 5x - 14)$

$= 6(x - 7)(x + 2)$

f is a decreasing function.

$\therefore f'(x) < 0, \therefore 6(x - 7)(x + 2) < 0$

Case (i)$x - 7 > 0 and\ x + 2 < 0, \therefore x > 7 and x < -2$.This is not possible.

Case(ii) $x - 7 < 0 and\ x + 2 > 0, \therefore x < 7 and x > -2$,

$\therefore \boldsymbol{-2} < x < 7$

Q.10 $I = \int \frac{dx}{1+3sin^2x} = \int \frac{sec^2xdx}{sec^2x+3tan^2x} = \int \frac{sec^2xdx}{1+tan^2x+3tan^2x}$

$= \int \frac{sec^2xdx}{1+4tan^2x} = \frac{1}{4}\int \frac{sec^2xdx}{\left(\frac{1}{2}\right)^2+tan^2x}$

Let tanx = t,$\therefore sec^2xdx = dt$

$\therefore I = \frac{1}{4}\int \frac{dt}{\left(\frac{1}{2}\right)^2+t^2} = \frac{1}{4} \times \frac{1}{(1/2)} \tan^{-1}\frac{t}{(1/2)} + c$

$= \frac{1}{2}\tan^{-1} 2t + c = \boldsymbol{\frac{1}{2}\tan^{-1}(2tanx) + c}$

Q.11 For the following probability distribution of X

X	0	1	2	3	4
P(X=x)	2k	5k	4k	3k	2k

Since it is a p.m.f.

2k + 5k + 4k + 3k + 2k = 1,$\boldsymbol{k = \frac{1}{16}}$

(ii) $P(X \geq 2) = 4k + 3k + 2k = 9k = \boldsymbol{\frac{9}{16}}$

Q.12 Let the radius and height of the cylinder be ‘r’ and ‘h’.

$r + h = 6, \therefore h = 6 - r$

Volume,$V = \pi r^2h = \pi r^2(6 - r) = \pi(6r^2 - r^3)$

Differentiating w.r.t.’r’

$\frac{dV}{dr} = \pi(12r - 3r^2)$

For maximum volume: $\frac{dV}{dr} = 0$, $\therefore \pi(12r - 3r^2) = 0, \therefore r = 4$

$\therefore h = 2$

$\frac{d^2V}{dr^2} = \pi(12 - 6r)$

For $r = 4, \frac{d^2V}{dr^2} = -12\pi < 0, \therefore$Maxima

Volume is maximum when $r = 4, h = 2$

Maximum volume is $\boldsymbol{32\pi cu. units}$

Q.13 $y = Ae^{4x} + Be^{-4x}$

Differentiating w.r.t.x

$\frac{dy}{dx} = 4Ae^{4x} - 4Be^{-4x}$

Differentiating w.r.t.x

$\frac{d^2y}{dx^2} = 16Ae^{4x} + 16Be^{-4x} = 16y$

$\therefore \boldsymbol{\frac{d^2y}{dx^2} = 16y}$ is the required differential equation.

Q.14 Let X = coin shows tail

$P(X = x) = n_{C_x} p^x q^{n-x}$

Here n = 6, p = 0.5, q = 0.5

$P(X = 2) = 6_{C_2}(0.5)^2(0.5)^4 = \frac{6\times5}{1\times2} \times \frac{1}{64} = \boldsymbol{\frac{15}{64}} = \boldsymbol{0.2344}$

Section C

Attempt any Eight

Q.15 In ΔABC, a = 4, b = 5, c = 3

$s = \frac{a+b+c}{2} = \frac{4+5+3}{2} = 6$

s – a = 2, s – b = 1, s – c = 3

(a) $sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{1(3)}{15}} = \boldsymbol{\frac{1}{\sqrt{5}}}$

(b)$cosA = \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+3^2-4^2}{2(5)(3)} = \frac{25+9-16}{30} = \frac{18}{30} = \boldsymbol{\frac{3}{5}}$

(c) A$(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6 \times 2 \times 1 \times 3} = \boldsymbol{6sq. units}$

Q.16 Let OA (or OB) passing through O,origin form equilateral triangle with the line x + y = 10. Let slope of OA (or OB) be $m_1$and of the line x + y =10 be m_2. $\therefore m_2 = -1$

$\tan 60 = \left|\frac{m_1-m_2}{1+m_1m_2}\right|, \therefore \sqrt{3} = \left|\frac{m_1+1}{1-m_1}\right|,$

$squaring, 3(1 - m_1)^2 = (m_1 + 1)^2$

$\therefore 3 - 6m_1 + 3{m_1}^2 = {m_1}^2 + 2m_1 + 1, \therefore {m_1}^2 - 4m_1 + 1 = 0$

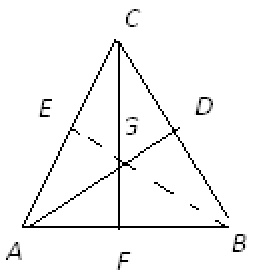
Since OA passes through the origin, $m_1 = \frac{y}{x}$

$\therefore \left(\frac{y}{x}\right)^2 - 4\frac{y}{x} + 1 = 0, x^2 - 4xy + y^2 = 0$is the pair of lines forming an equilateral

triangle with the line x + y = 10.
Let p be the distance of the origin from the line x + y = 10.
∴ p = height of the equilateral triangle.
$p = \left|\frac{0+0-10}{\sqrt{1+1}}\right| = \frac{10}{\sqrt{2}}$
Area of the equilateral triangle is $\frac{p^2}{\sqrt{3}} = \frac{\mathbf{50}}{\sqrt{\mathbf{3}}}\boldsymbol{sq.units}$

Q.17 Let $\frac{x-1}{2} = \frac{y}{1} = \frac{z}{2} = t$
Any point on the line is (2t+1, t,2t). Let M be the foot of the perpendicular. Let M = (2t+1, t,2t)Let P = (2,4,-1).
Drs of the line are 2,1,2 and drs of PM are $2t-1, t-4, 2t+1$
PM is perpendicular to the line.
$\therefore 2(2t-1)+1(t-4)+2(2t+1)=0, \therefore 4t-2+t-4+4t+2=0, \therefore 9t = 4, t = \frac{4}{9}$
$\therefore \boldsymbol{M} = (\frac{\mathbf{17}}{\mathbf{9}}, \frac{\mathbf{4}}{\mathbf{9}}, \frac{\mathbf{8}}{\mathbf{9}})$

Q.18 Let A, B, C be the vertices of the triangle. Let D, E, F be the midpoints of sides BC, CA, AB respectively. Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}, \bar{f}$ be the position vectors of points A, B, C, D, E, F w.r.t. some fixed origin O. Therefore, by midpoint formula,
$\bar{d} = \frac{\bar{b}+\bar{c}}{2}, \bar{e} = \frac{\bar{a}+\bar{c}}{2}, \bar{f} = \frac{\bar{a}+\bar{b}}{2}$



$\therefore 2\bar{d} = \bar{b}+\bar{c}, 2\bar{e} = \bar{a}+\bar{c}, 2\bar{f} = \bar{a}+\bar{b}$
$\therefore 2\bar{d}+\bar{a} = \bar{a}+\bar{b}+\bar{c}, 2\bar{e}+\bar{b} = \bar{a}+\bar{b}+\bar{c}, 2\bar{f}+\bar{c} = \bar{a}+\bar{b}+\bar{c}$
$\therefore \frac{2\bar{d}+\bar{a}}{3} = \frac{2\bar{e}+\bar{b}}{3} = \frac{2\bar{f}+\bar{c}}{3} = \frac{\bar{a}+\bar{b}+\bar{c}}{3} = \bar{g}$ where $\bar{g}$ is the position vector of the point G. ∴G lies on all the three medians. ∴ The Medians are concurrent.

Q.19 Let $\bar{a} = -2\bar{i}+\bar{j}-\bar{k}, \bar{b} = -3\bar{i}-4\bar{j}+\bar{k}$
A vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\bar{a}\times\bar{b}$
$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & 1 & -1 \\ -3 & -4 & 1 \end{vmatrix} = -3\bar{i}+5\bar{j}+11\bar{k}$$
Required direction ratios are **-3,5,11**

Q.20 Let the equation of the plane be $ax+by+cz+d=0$
Since the plane is parallel to X-axis, its normal is perpendicular to X-axis.

Let the drs of the normal be a, b, c.
Drs of the X-axis are 1,0,0
$\therefore a(1) + b(0) + c(0) = 0, a = 0$
Equation of the plane is $by + cz + d = 0$
$(2,3,1), (4,-5,3)$ lie on it.
$\therefore 3b + c + d = 0 \ldots..(1)\ and - 5b + 3c + d = 0 \ldots..(2)$
Subtracting (2) from (1) we get $8b - 2c = 0, \therefore c = 4b$
In (1)$3b + 4b + d = 0, \therefore d = -7b$
$\therefore$Equation of the plane is $by + 4bz - 7b = 0, \therefore \boldsymbol{y + 4z = 7}$
Vector equation is $\boldsymbol{\bar{r}.(\bar{j} + 4\bar{k}) = 7}$

Q.21 $x = acos^3t, y = asin^3t$
Differentiating w.r.t.'t'
$\frac{dx}{dt} = -3acos^2tsint, \frac{dy}{dt} = 3asin^2tcost$
$\therefore \frac{dy}{dx} = \frac{\left({dy}/{dt}\right)}{\left({dx}/{dt}\right)} = \frac{3asin^2tcost}{-3acos^2tsint} = -\frac{sint}{cost} \ldots.(1)$
$\frac{y}{x} = \frac{asin^3t}{acos^3t} = \left(\frac{sint}{cost}\right)^3,$
$\therefore \frac{sint}{cost} = \left(\frac{y}{x}\right)^{\frac{1}{3}}$
In (1), $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$.Hence proved.

Q.22 $I = \int \frac{1}{3+2sinx+cosx}dx$
Let $tan\frac{x}{2} = t, \therefore dx = \frac{2dt}{1+t^2}, sinx = \frac{2t}{1+t^2}, cosx = \frac{1-t^2}{1+t^2}$
$\therefore I = \int \frac{\frac{2dt}{1+t^2}}{3+2\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{2dt}{3+3t^2+4t+1-t^2} = \int \frac{2dt}{2t^2+4t+4}$
$= \int \frac{dt}{t^2+2t+2} = \int \frac{dt}{t^2+2t+1+1} = \int \frac{dt}{(t+1)^2+(1)^2}$
$= \tan^{-1}\left(\frac{t+1}{1}\right) + c$ ....$\{\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\}$
$= \tan^{-1}(t+1) + c = \boldsymbol{\tan^{-1}\left(tan\left(\frac{x}{2}\right) + 1\right) + c}$

Q.23 $\frac{dy}{dx} = y + 2x$
$\frac{dy}{dx} - y = 2x$. This is of the type $\frac{dy}{dx} + Py = Q$,where P and Q are functions of x.
Here P = -1, $\therefore I.F. = e^{\int Pdx} = e^{\int -1dx} = e^{-x}$
Solution is
$y(I.F.) = \int Q(I.F.)dx + c$
$\therefore y(e^{-x}) = \int 2x(e^{-x})dx + c$
$\therefore y(e^{-x}) = 2[-xe^{-x} - \int(1)(-e^{-x})\,dx] + c$
$\therefore y(e^{-x}) = -2xe^{-x} - 2e^{-x} + c$
$\therefore y = -2x - 2 + ce^x, \therefore \boldsymbol{y + 2x + 2 = ce^x}$

Since the tangent passes through the origin,
$0+0+2=c, \therefore c=2$
The equation of the curve is
$\boldsymbol{y+2x+2=2e^{x}}$

Q.24 $\frac{dV}{dt}=8, r=2, V=\frac{4}{3}\pi r^3, \frac{dV}{dt}=\frac{4}{3}\pi 3r^2\frac{dr}{dt}, \therefore 8=4\pi(2)^2\left(\frac{dr}{dt}\right)$
$\boldsymbol{\frac{dr}{dt}=\frac{1}{2\pi}cm/sec}$ **.** Surface area, $A=4\pi r^2, \frac{dA}{dt}=4\pi(2r)\frac{dr}{dt}=8\pi(2)\left(\frac{1}{2\pi}\right)=\boldsymbol{8cm^2/sec}$

Q.25 X~ B (n, p)
(i)E(X) = np = 5, ∴15p = 5, ∴**p** = $\boldsymbol{\frac{1}{3}}$.q = 1 – p = 1 - $\frac{1}{3}=\frac{2}{3}$
Var(X) = npq =$15\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)=\boldsymbol{\frac{10}{3}}$
(ii) E(X) = 9, Var(x) = 4.5
∴ np = 9 and npq = 4.5,$q=\frac{4.5}{9}=\frac{1}{2}$,p = 1 – q = 1 - $\frac{1}{2}$,∴**p** = $\boldsymbol{\frac{1}{2}}$
$n\left(\frac{1}{2}\right)=9$,∴**n = 18**

Q.26

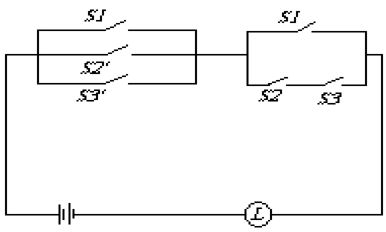
X = x	0	1	2	3	4
P(X = x)	0.2	0.1	0.2	0.4	0.1

Expected value, E(x) = $\sum x_iP_i=0(0.2)+1(0.1)+2(0.2)+3(0.4)+4(0.1)=0+0.1+0.4+1.2+0.4=\boldsymbol{2.1}$
E$(x^2)=x^2P(x)=0^2(0.2)+1^2(0.1)+2^2(0.2)+3^2(0.4)+4^2(0.1)=0+0.1+0.8+3.6+1.6=\boldsymbol{6.1}$
Variance(x) = E$(x^2)-$ E$(x)^2=6.1-(2.1)^2$
$=6.1-4.41=\boldsymbol{1.69}$

Section D

Attempt any five

Q.27



Let p: switch S_1 is closed, q: switch S_2 is closed and r: switch S_3 is closed.
Symbolic form: $(p\vee\sim q\vee\sim r)\wedge[p\vee(q\wedge r)]$

p	q	r	$\sim q$	$\sim r$	$p\vee\sim q\vee\sim r$ (1)	$q\wedge r$	$p\vee(q\wedge r)$ (2)	(1)∧(2)
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	F	T	T
T	F	T	T	F	T	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	F	F	T	T	F

F	T	F	F	T	T	F	F	F
F	F	T	T	F	T	F	F	F
F	F	F	T	T	T	F	F	F

Q.28 $|A| = -13 + 6 + 6 = -1$

$A^{-1} = \frac{1}{|A|}(adjoint\ A)$

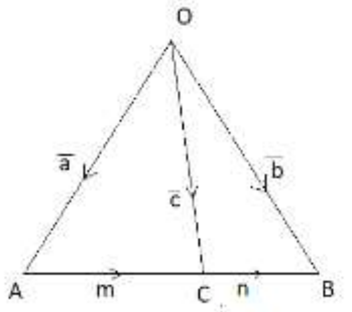
Minors: $M_{11} = -13, M_{12} = -3, M_{13} = 2, M_{21} = 2, M_{22} = 1,$
$M_{23} = 0\ ,\ M_{31} = 7, M_{32} = 2, M_{33} = -1$

Co-factors: $A_{11} = -13, A_{12} = 3, A_{13} = 2, A_{21} = -2, A_{22} = 1,$
$A_{23} = 0, A_{31} = 7, A_{32} = -2, A_{33} = -1$

AdjointA = $\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$

$$A^{-1} = -\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \mathbf{13} & \mathbf{2} & \mathbf{-7} \\ \mathbf{-3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{-2} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Q.29



$\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C w.r.t. origin and point C divides AB internally in the ratio of m:n

Now,$\frac{AC}{CB} = \frac{m}{n}, \therefore n.AC = m.CB$

Since AC and CB are in the same direction and are collinear,

$n\overline{AC} = m\overline{CB}, \therefore n(\bar{c} - \bar{a}) = m(\bar{b} - \bar{c})$

$n\bar{c} - n\bar{a} = m\bar{b} - m\bar{c}, \therefore (m+n)\bar{c} = m\bar{b} + n\bar{a}, \therefore \bar{c} = \frac{m\bar{b}+n\bar{a}}{m+n}$

A(2,-1,3) and B(-5,2,-5)

$\bar{a} = 2\bar{\imath} - \bar{\jmath} + 3\bar{k}, \bar{b} = -5\bar{\imath} + 2\bar{\jmath} - 5\bar{k}$

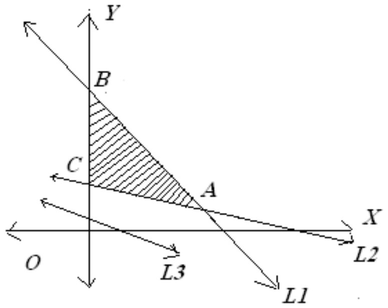
$m: n = 3: 2$

$\therefore \bar{c} = \frac{3(-5\bar{\imath}+2\bar{\jmath}-5\bar{k})+2(2\bar{\imath}-\bar{\jmath}+3\bar{k})}{3+2} = -\frac{\mathbf{11}}{\mathbf{5}}\bar{\boldsymbol{\imath}} + \frac{\mathbf{4}}{\mathbf{5}}\bar{\boldsymbol{\jmath}} - \frac{\mathbf{9}}{\mathbf{5}}\bar{\boldsymbol{k}}$

Q.30

Inequalities	Equalities	Points to be plotted
$x + y \leq 8$	$x + y = 8$	(8,0),(0,8)…L1
$x + 4y \geq 12$	$x + 4y = 12$	(12,0),(0,3)…L2

$5x + 8y \geq 20$	$5x + 8y = 20$	(4,0),(0,2.5)…L3
$x \geq 0$	x = 0	
$y \geq 0$	y = 0	



Shaded portion is the solution set.

Points	z = 30x + 20y
$A\left(\frac{20}{3}, \frac{4}{3}\right)$	$z = \frac{680}{3} = 226.7$
B(0,8)	z = 160
C(0,3)	z = 60

z is minimum at C (0,3) and the minimum value is 60.

Q.31 Let small increment in x be δx $\therefore$ Let u increase by δu and y by δy.

Consider $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x}$. Taking limit as $\delta x \to 0$ on both sides,

$\lim_{\delta x \to 0} \frac{\partial y}{\partial x} = \lim_{\delta x \to 0} \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x}$,

as $\delta x \to 0, \delta u \to 0$ $\therefore$ $\lim_{\delta x \to 0} \frac{\partial y}{\partial x} = \lim_{\delta u \to 0} \frac{\partial y}{\partial u} \cdot \lim_{\delta x \to 0} \frac{\partial u}{\partial x}$

Since y is a differentiable function of u and u is a differentiable function of x,

$\lim_{\delta u \to 0} \frac{\partial y}{\partial u} = \frac{dy}{du}$ and $\lim_{\delta x \to 0} \frac{\partial u}{\partial x} = \frac{du}{dx}$.

$\therefore$ $\lim_{\delta x \to 0} \frac{\partial y}{\partial x} = \frac{dy}{du} \cdot \frac{du}{dx}$. Since limits on R.H.S. exists, Limits on L.H.S. exist and is $\frac{dy}{dx}$.

$\therefore$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. Hence proved.

$y = sin^2(x + 3)$

Let $\sin(x + 3) = u, y = u^2$

$\frac{dy}{du} = 2u, \frac{du}{dx} = cos(x + 3)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \mathbf{2\sin(x + 3)\cos(x + 3)}$

Q.32 $I = \int e^{3x} sin2x \, dx$

Integrating by parts,

$I = \int e^{3x} sin2x \, dx$

$= -e^{3x} \frac{cos2x}{2} + \int 3e^{3x} \frac{cos2x}{2} dx$

$= -e^{3x} \frac{cos2x}{2} + \frac{3}{2} e^{3x} \frac{sin2x}{2} - \frac{3}{2} \int 3e^{3x} \frac{sin2x}{2} dx$

$I = -e^{3x} \frac{cos2x}{2} + \frac{3}{2} e^{3x} \frac{sin2x}{2} - \frac{9}{4} I$

$\frac{13}{4}I = \frac{1}{4}[-2e^{3x}cos2x + 3e^{3x}sin2x]$

$\therefore I = \frac{e^{3x}}{13}[\mathbf{3sin2x - 2cos2x}] + \boldsymbol{c}$

Q.33 If f is odd f(-x) = - f(x) and if f is even f(-x) = f(x)….(1)

L.H.S. = $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$……

$\left\{using, \int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx\, , for a < c < b\right\}$

∴ L.H.S = $I_1 + \int_{0}^{a} f(x)dx$…………..(2)

$I_1 = \int_{-a}^{0} f(x)dx$, let x = - t,∴ dx = - dt, also when x = - a, t = a and when x = 0, t = 0.

$\therefore I_1 = -\int_{a}^{0} f(-t)dt = \int_{0}^{a} f(-t)dt$….$\left\{\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx\right\}$

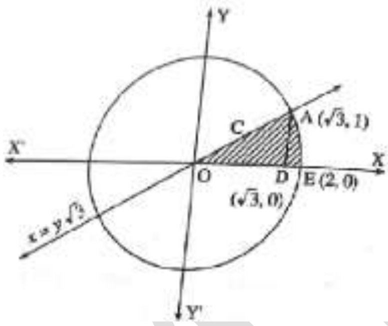
$\therefore I_1 = \int_{0}^{a} f(-x)dx$………$\left\{\int_{a}^{b} f(t)dt = \int_{a}^{b} f(x)dx\right\}$

∴ In (1), L.H.S = $\int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx = \int_{0}^{a}[f(-x) + f(x)]dx$

$\therefore \int_{-a}^{a} f(x)dx = 0 \qquad if\ f\ is\ odd$

$= 2\int_{0}^{a} f(x)dx \qquad if\ f\ is\ even$.As required.

Q.34



To get the point of intersection of the circle with the line, solving the two equations simultaneously,

$\left(y\sqrt{3}\right)^2 + y^2 = 4, y^2 = 1, y = \pm 1$

$x = \pm\sqrt{3}$

A is the point in the first quadrant, $\therefore A\left(\sqrt{3}, 1\right)$

Draw AD perpendicular to OE.

$E(2,0)$

Area of the shaded portion = Area under the line OA +Area under the circle (arcAE)

$\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}dx$

$= \left[\frac{x^2}{2\sqrt{3}}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$

$= \frac{\sqrt{3}}{2} + \left(0 + 2.\frac{\pi}{2}\right) - \left(\frac{\sqrt{3}}{2} + 2.\frac{\pi}{3}\right) = \boldsymbol{\frac{\pi}{3} sq.units}$